In this report, we implemented a simple linear regression of the Dow Jones Industrial Average (“DJIA”) index over the S&P500 (“S&P”) using daily and annual log returns.

Our key finding is that the return characteristics of the DJIA and S&P are not different from each other at the 5% level of significance.

**1. Task 3 – Regression using daily log returns**

**1.1 Estimate of key statistics \hat{a} \hat{b} and \hat{\sigma\_u}**

\hat{a} 0.000046

\hat{b} 0.943110

\hat{\sigma\_u} 0.002877

The regression results in an intercept of 0.00464% and a slope of 0.9431.

The positive intercept indicates that the DJIA has a small positive daily excess returns on average as compared to the S&P.

The slope of 0.9431 indicates that the DJIA is slightly less volatile than the S&P. In this context, the slope is interpreted as Beta – the sensitivity of DJIA’s returns to the S&P’s returns. Assuming investors are risk-averse, a lower Beta is preferred for the same level of return because a lower Beta asset will have more consistency in its returns.

Combining these two measures together, it suggests that the DJIA has a superior risk-adjusted return as compared to the S&P.

**1.2 t-test for Null Hypothesis a=b=0 at 5% significance**

The t-test statistic for \hat{a} is 1.486364

The t-test statistic for \hat{b} is 339.97

Upper and lower critical values: ± 1.960243

We conduct the t-test to deduce if our estimates of \hat{a} and \hat{b} are significant at the 5% level.

Our sample size is 8500 observations, leading to 8498 degrees of freedom

The test statistic for \hat{a} falls within the critical values, and thus we cannot reject the null hypothesis that \hat{a}=0. The indication of DJIA having higher daily returns than the S&P is therefore not significant at the 5% level.

However, the test statistic for \hat{b} falls outside the critical values, and thus we reject the null hypothesis that \hat{b}=0 and conclude that there is a linear relationship between DJIA and S&P at the 5% significance level.

**1.3 R^2 and Adjusted R^2 values**

R^2 0.931512

Adjusted R^2 0.931504

The R^2 value is very high, further supporting our previous point that there exists a linear relationship between the daily returns of the DJIA and the S&P.

**1.4 Jarque-Bera test statistic for the residuals**

JB statistic (\hat{u}) 25434.27

JB test critical value 5.99146

The JB test statistic exceeds the critical value by a huge margin, strongly indicating that the residuals are not normally distributed. This is due to regression outliers that were a result of extreme market conditions, for example the huge one-day drop on 19-Oct-1987.

**2. Task 4 – Regression using annual log returns**

**2.1 Estimate of key statistics \hat{a} \hat{b} and \hat{\sigma\_u}**

\hat{a} 0.019784

\hat{b} 0.842545

\hat{\sigma\_u} 0.037969

The regression results in an intercept of 1.978% and a slope of 0.8425.

The positive intercept indicates that the DJIA has a small positive daily excess returns on average as compared to the S&P.

The slope of 0.8425 indicates that the DJIA is slightly less volatile than the S&P.

Combining these two measures together, it suggests that the DJIA has a superior risk-adjusted return as compared to the S&P.

**2.2 t-test for Null Hypothesis a=b=0 at 5% significance**

The t-test statistic for \hat{a} is 2.649890

The t-test statistic for \hat{b} is 20.436400

Upper and lower critical values: ± 2.042272

We conduct the t-test to deduce if our estimates of \hat{a} and \hat{b} are significant at the 5% level.

Our sample size is 32 observations, leading to 30 degrees of freedom

The test statistic for \hat{a} falls outside the critical values, and thus we reject the null hypothesis that \hat{a}=0. The t-test at 5% significance concludes that the DJIA has higher annual returns as compared to the S&P.

The test statistic for \hat{b} also falls outside the critical values, and thus we reject the null hypothesis that \hat{b}=0 and conclude that there is a linear relationship between the annual returns of the DJIA and the S&P at the 5% significance level.

**2.3 R^2 and Adjusted R^2 values**

R^2 0.932983

Adjusted R^2 0.930749

The R^2 value is very high, further supporting our previous point that there exists a linear relationship between the annual returns of the DJIA and the S&P.

**2.4 Jarque-Bera test statistic for the residuals**

JB statistic (\hat{u}) 1.046372

JB test critical value 5.99146

The JB test statistic falls within the critical value, indicating that the regression residuals are normally distributed.